

Chapter 9

Exercise 9A

- 1 a $8x^7$
 b $-4x^3$
 c 1
 d $-\frac{1}{x^2}$
 e $-\frac{6}{x^7}$
 f $\frac{9}{x^{10}}$
 g 0
 h $12x^2$
 i $6x^7$
 j $-\frac{18}{x^3}$
 k 0
 l $\frac{1}{2x^4}$
 m $\frac{3\sqrt{x}}{2}$
 n $\frac{5x^{\frac{2}{3}}}{3}$
 o $\frac{7x^{\frac{2}{5}}}{5}$
 p $\frac{1}{2\sqrt{x}}$
 q $\frac{1}{4x^{\frac{3}{4}}}$
 r $\frac{2}{3x^{\frac{1}{3}}}$
 s $-\frac{1}{2x^{\frac{3}{2}}}$
 t $\frac{2}{3x^{\frac{5}{3}}}$
 u $\frac{4}{\sqrt{x}}$
 v $-\frac{2}{x^{\frac{4}{3}}}$
 w $-\frac{5\sqrt{x}}{2}$
 x 0
 y $-\frac{3}{4x^{\frac{11}{4}}}$
- 2 a $2 + 2x$
 b $-4x - 8$
 c 3
 d $3x^2 - 8x + 8$
 e $x^2 + 4x - 12$
 f $4x^3 - 6x^2 + 6x - 1$
 g $\frac{12}{x^7} - 15x^2$

- h $\frac{6}{x^7} - 2$
 i $2x^3 + \frac{6}{x^3} + 2$
 j $\frac{3x^4}{2} + \frac{1}{x^2}$
- 3 a $-\frac{1}{2x^{\frac{3}{2}}} + \frac{1}{2\sqrt{x}}$
 b $18\sqrt{x} - \frac{5x^{\frac{3}{2}}}{2}$
 c $18x^2 - \frac{1}{\sqrt{x}}$
 d $\frac{1}{x^{\frac{4}{3}}} + \frac{x^{\frac{1}{2}}}{2}$
 e $-\frac{2}{x^{\frac{3}{2}}} - \frac{8}{x^7} - 8$
 f $\frac{2}{x^{\frac{3}{4}}} - \frac{4}{x^7}$
 g $\frac{6}{x^{\frac{1}{4}}} + \frac{1}{5x^{\frac{3}{2}}}$
 h $\frac{1}{x^{\frac{5}{4}}} - \frac{8}{x^7} - 8$
 i $\frac{1}{5x^{\frac{3}{4}}} + \frac{1}{4x^{\frac{6}{5}}}$
- 4 a $3x^2 - 8x + 1$
 b $\frac{4x^{\frac{1}{3}}}{3}$
 c $\frac{3}{\sqrt{x}}$
 d $4x - 3$
 e $\frac{3}{4x^{\frac{1}{4}}} - 8$
- 5 a $4p^3 - 12p$
 b $15p^4 + \frac{2}{p^3}$
 c $3w^2 - 3$
 d $5 - 30t^2$
 e $12\sqrt{t} + \frac{2}{t^{\frac{3}{2}}}$
 f $\frac{8}{t^{\frac{1}{3}}} - 1$
 g $-\frac{24}{t^5} - 8$
- 6 a $-\frac{8}{x^3} - 5$
 c $\frac{12}{x^5} - 5$
 d $x^{\frac{1}{3}}$
 e $\frac{2}{x^2} + \frac{6}{x^3}$
 f $\frac{2}{x^{\frac{1}{3}}} + \frac{2}{x^{\frac{4}{3}}}$

Exercise 9B

- 1 a $2x^2 + 5x - 12$
 b $4x^2 - 7x + 3$
 c $2x^3 - 11x^2 + 17x - 6$
 d $x^3 - 5x^2 + 2x + 8$
 e $x^3 - 3x + 2$
 f $4x^3 - 4x^2 + x$
 g $x^3 - 4x^2 + x$

- 2 a $3x^{-1}$
 b $8x^{-3}$
 c $5x^{-6}$
 d $\frac{1}{3}x^{-2}$
 e $\frac{1}{6}x^{-4}$
 f $\frac{2}{5}x^{-1}$
 g $\frac{4}{3}x^{-9}$
 h $\frac{1}{2}x^{-7}$
 i $3x^{-\frac{1}{2}}$
 j $x^{-\frac{3}{4}}$
 k $5x^{-\frac{4}{3}}$
 l $\frac{4}{5}x^{-\frac{3}{2}}$
 m $2x^{-\frac{5}{8}}$
 n $2x^{\frac{1}{2}}$
 o $5x^{-\frac{1}{2}}$
 p $\frac{1}{2}x^{-\frac{1}{2}}$
 q $\frac{3}{4}x^{\frac{1}{5}}$
 r x^{-4}
 s $x^{-\frac{1}{6}}$
 t $\frac{5}{3}x^{-\frac{1}{4}}$
 u $6x^2$
 v $2x^{-3}$
 w $\frac{3}{8}x^{-5}$
 x $4x^{-\frac{1}{2}}$
 y $\frac{1}{5}x^{\frac{4}{3}}$
 z $\frac{7}{4}x^{-2}$

- 3 a $x^2 + 2x - 15$
 $2x + 2$
 b $8x^2 - 10x - 3$
 $16x - 10$
 c $x^3 + x^2 - 6x$
 $3x^2 + 2x - 6$
 d $x^3 + 5x^2 + 2x - 8$
 $3x^2 + 10x + 2$
 e $2x^4 - 4x^3 + 2x^2$
 $8x^3 - 12x^2 + 4x$
 f $x^3 - 5x^2 + 3x + 9$
 $3x^2 - 10x + 3$
 g $x^3 + 3x^2 - 6x - 8$
 $3x^2 + 6x - 6$

- 4 a $5x^{-2}$
 $-\frac{10}{x^3}$
 b $7x^{-4}$
 $-\frac{28}{x^5}$
 c $\frac{1}{2}x^{-3}$
 $-\frac{3}{2x^4}$
 d $\frac{1}{6}x^{-2}$
 $-\frac{1}{3x^3}$
 e $4x^3 - 2x^{-5}$
 $\frac{10}{x^6} + 12x^2$
 f $\frac{4}{3}x^{-1}$
 $-\frac{4}{3x^2}$
 g $8x + 5 - x^{-2}$
 $\frac{2}{x^3} + 8$
 h $4x^{-3} - 3x^{-1}$
 $-\frac{12}{x^4} + \frac{3}{x^2}$
 i $\frac{3}{2}x^{-4} - 5x - 6$
 $-5 - \frac{6}{x^5}$

- 5 a $8x^{\frac{1}{2}}$
 $\frac{4}{\sqrt{x}}$
- b $x^{\frac{2}{3}}$
 $\frac{2}{3\sqrt[3]{x}}$
- c $12x^{\frac{3}{4}}$
 $\frac{9}{\sqrt[4]{x}}$
- d $4x^{\frac{5}{2}}$
 $10\sqrt[2]{x^3}$
- e $x^{\frac{7}{2}}$
 $\frac{7}{2}\sqrt{x^5}$
- f $x^{-\frac{1}{2}}$
 $-\frac{1}{\sqrt{x^3}}$
- g $x^{-\frac{3}{2}}$
 $-\frac{3}{2\sqrt[2]{x^5}}$
- h $4x^{-\frac{1}{4}}$
 $-\frac{1}{\sqrt[4]{x^5}}$
- i $10x^{-\frac{3}{2}}$
 $-\frac{15}{\sqrt[2]{x^5}}$
- j $9x^{-\frac{5}{6}}$
 $-\frac{15}{2\sqrt[6]{x^{11}}}$
- k $\frac{3}{2}x^{\frac{4}{9}}$
 $\frac{2}{3\sqrt[9]{x^5}}$
- l $\frac{1}{8}x^{-\frac{6}{5}}$
 $-\frac{3}{20\sqrt[5]{x^{11}}}$
- m $\frac{5}{2}x^{-\frac{2}{3}}$
 $-\frac{5}{3\sqrt[3]{x^5}}$
- 6 a $x - 4x^{-1}$
 $\frac{4}{x^2} + 1$

- b $3 - 5x^{-1}$
 $\frac{5}{x^2}$
- c $4x^{-1} - x^2$
 $-\frac{4}{x^2} - 2x$
- d $1 + 5x^{-1} - 3x^{-2}$
 $\frac{6}{x^3} - \frac{5}{x^2}$
- e $\frac{1}{2}x^{-3} - \frac{1}{2}x$
 $-\frac{1}{2} - \frac{3}{2x^4}$
- f $-\frac{1}{3}x^{-3} - \frac{1}{2}x^{-2} + \frac{1}{6}x^{-1}$
 $\frac{1}{x^4} + \frac{1}{x^3} - \frac{1}{6x^2}$
- g $1 + 4x^{-2} + 5x^{-1}$
 $-\frac{8}{x^3} - \frac{5}{x^2}$
- h $5x^{-1} + 3x^{-2} - 2$
 $-\frac{6}{x^3} - \frac{5}{x^2}$
- i $4 - 4x^{-1} - 7x^{-2} - 2x^{-3}$
 $\frac{6}{x^4} + \frac{14}{x^3} + \frac{4}{x^2}$

7 Please note that these answers are just one of many possible “forms” that the correct answers may take.

- a $-\frac{3}{2\sqrt{x}} + \frac{3\sqrt{x}}{2}$
- b $\frac{9}{x^4}$
- c $\frac{1}{2\sqrt{x^3}} - \frac{1}{\sqrt{x}}$
- d $-\frac{1}{x^{\frac{3}{2}}} + \frac{4}{x^2} - \frac{3}{x^{\frac{5}{2}}}$
- e $-\frac{3}{2x^{\frac{5}{2}}} + 3\sqrt{x}$
- f $-\frac{18}{x^3} + \frac{2x}{9}$

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$$\begin{aligned} \text{a} & \quad -\frac{1}{x^{\frac{3}{2}}} - \frac{5x^{\frac{3}{2}}}{2} \\ \text{b} & \quad -\frac{1}{2x^{\frac{3}{2}}} - \frac{1}{2\sqrt{x}} + \frac{3\sqrt{x}}{8} \\ \text{c} & \quad -\frac{4}{x^5} + \frac{9x}{2x^4} - \frac{1}{x^3} \\ \text{d} & \quad \frac{-5}{2x^{\frac{3}{2}}} + \frac{2}{x^2} + \frac{15}{x^{\frac{5}{2}}} \\ \text{e} & \quad -\frac{25}{6x^{\frac{11}{3}}} \end{aligned}$$

Exercise 9C

$$1 \quad \text{Lim}_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\text{Lim}_{h \rightarrow 0} \frac{x+h-x}{h}$$

$$\text{Lim}_{h \rightarrow 0} \frac{h}{h}$$

1

$$2 \quad \text{Lim}_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\text{Lim}_{h \rightarrow 0} \frac{cx+ch-cx}{h}$$

$$\text{Lim}_{h \rightarrow 0} \frac{ch}{h}$$

c

$$3 \quad \text{a} \quad \text{Lim}_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$\text{Lim}_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$\text{Lim}_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$\text{Lim}_{h \rightarrow 0} 3x^2 + 3hx + h^2$$

$$= 3x^2$$

$$\text{b} \quad \text{Lim}_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$\text{Lim}_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$$

$$\text{Lim}_{h \rightarrow 0} h^3 + 4h^2x + 6hx^2 + 4x^3$$

$$= 4x^3$$

$$4 \quad (x+h)^n = x^n + nx^{n-1}h + (n-1)x^{n-1}h^2$$

$$+ (n-2)x^{n-2}h^2 + \dots + 1x^0h^n$$

numerator of first principle quotient:

$$\begin{aligned} (x+h)^n - x^n &= x^n + nx^{n-1}h + (n-1)x^{n-1}h^2 \\ &+ (n-2)x^{n-2}h^2 + \dots + 1x^0h^n - x^n \end{aligned}$$

denominator = h

quotient:

$$nx^{n-1} + (n-1)x^{n-1}h + (n-2)x^{n-2}h^2 + \dots + h^{n-1}$$

$$\text{Lim}_{h \rightarrow 0} (nx^{n-1} + (n-1)x^{n-1}h$$

$$+ (n-2)x^{n-2}h^2 + \dots + h^{n-1}) = nx^{n-1}$$

5 still missing

$$6 \quad \text{a} \quad \frac{x-(x+h)}{x(x+h)} = -\frac{h}{x(x+h)}$$

$$\text{b} \quad \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{-h}{x(x+h)}}{h} = -\frac{1}{x(x+h)}$$

$$\text{c} \quad \text{Lim}_{h \rightarrow 0} \left(-\frac{1}{x(x+h)} \right) = \frac{-1}{x^2}$$

$$\text{d} \quad \frac{\frac{k}{x+h} - \frac{k}{x}}{h} = \frac{\frac{-kh}{x(x+h)}}{h} = -\frac{k}{x(x+h)}$$

$$\text{Lim}_{h \rightarrow 0} \left(-\frac{k}{x(x+h)} \right) = \frac{-k}{x^2}$$

$$\text{e} \quad \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{\frac{-2xh-h^2}{x^2(x+h)^2}}{h} = \frac{-2x-h}{x^2(x+h)^2}$$

$$\text{Lim}_{h \rightarrow 0} \left(-\frac{-2x-h}{x^2(x+h)^2} \right) = \frac{-2x}{x^4} = -\frac{2}{x^3}$$

$$\frac{1}{(x+h)^3} - \frac{1}{x^3} = \frac{x^3 - (h^3 + 3h^2x + 3hx^2 + x^3)}{x^3(x+h)^3} = \frac{-(h^2 + 3hx + 3x^2)}{x^3(x+h)^3}$$

$$\text{Lim}_{h \rightarrow 0} \left(-\frac{(h^2 + 3hx + 3x^2)}{x^3(x+h)^3} \right) = \frac{-3x^2}{x^6} = -\frac{3}{x^4}$$

$$7 \quad \text{a} \quad (\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})$$

$$= (\sqrt{x+h})^2 - (\sqrt{x})^2$$

$$= h$$

$$\text{b} \quad \text{Lim}_{h \rightarrow 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right)$$

$$\text{Lim}_{h \rightarrow 0} \left(\frac{\frac{-h}{\sqrt{x+h} + \sqrt{x}}}{h} \right)$$

$$\text{Lim}_{h \rightarrow 0} \left(\frac{1}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$\text{c} \quad \frac{1}{2\sqrt{x}}$$

ANSWERS

$$8 \quad \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

$$\lim_{h \rightarrow 0} \left(\frac{-h}{\sqrt{x+h} + \sqrt{x}} \times \frac{1}{h\sqrt{x}\sqrt{x+h}} \right)$$

$$= \lim_{h \rightarrow 0} \left(\frac{-1}{\sqrt{x}(h+x)\sqrt{x}\sqrt{h+x}} \right) = \frac{-1}{2x^{\frac{3}{2}}}$$

$$9 \quad \lim_{h \rightarrow 0} \frac{g(x+h) + f(x+h) - (g(x) + f(x))}{h}$$

$$\lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} + \frac{f(x+h) - f(x)}{h}$$

$$= f'(x) + g'(x)$$

10 a Taking a larger Delta about centre of area of interest.

$$b \quad \frac{(x+h)^2 - (x-h)^2}{2h} = \frac{x^2 + 2xh + h^2 - (x^2 - 2xh + h^2)}{2h}$$

$$= \frac{4xh}{2h}$$

$$\lim_{h \rightarrow 0} = 2x$$

$$\frac{(x+h)^3 - (x-h)^3}{2h} = \frac{x^3 + 3xh^2 + 3x^2h + h^3 - (x^3 + 3xh^2 - 3x^2h - h^3)}{2h}$$

$$= \frac{2h^3 + 6hx^2}{2h}$$

$$\lim_{h \rightarrow 0} = 3x^2$$

Exercise 9D

- 1 a 4
b -2
c 2
d 3
- 2 a 10
b 5
c 0
d 1
e -6
f 4
- 3 a -2
b $-\frac{2}{9}$
c -8

- 4 a $\frac{3}{4}$
b $\frac{1}{2}$
c 6

5 -1

6 $\frac{1}{400}$

7 a -3

b -3

8 (-1, 8)

9 -2

10 a $x = 2$

$x = 4$

b $x = -2$

$x = \frac{2}{3}$

11 $k = -4$

(-2)

12 4

13 a = 38

b = 9

14 (-5, -12)

15 a 2

b Straight line gradient = $-\frac{1}{2}$,
 $m_1m_2 = -1$, so they are
perpendicular.

16 a (6, 5)

b (2, 5)

c (4, 0)

17 a $3(x-1)^2 + 1$

b $3x^2 - 6x + 2 = 3(x-1)^2 - 1$

Minimum value of expression is -1.

18 note || means OR

$x < -2$ || $x > \frac{4}{3}$

19 a 0.3162

b 3

c (5.5, 4.25)

Exercise 9E

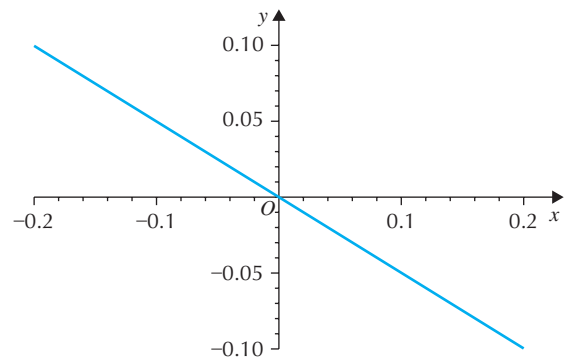
- 1 a $8\cos x$
 b $-3\sin x$
 c $-\cos x$
 d $\frac{1}{2}\cos x$
 e $-\frac{2}{3}\sin x$
 f $\frac{5}{8}\sin x$
 g $12x + 7\cos x$
 h $3\cos x - 7\sin x$
 i $6\cos x - \sin x$
 j $\cos x + \sin x$
 k $\sin x - \frac{6}{x^3}$
 l $\frac{4\cos x}{5} - \frac{3}{\sqrt{x}}$
 m $5\sin x - \frac{3}{4x^2}$
 n $9\cos x + 15x^2 + \frac{5}{3x^3}$
 o $3\sin x - \frac{2}{3x^3}$
 p $-\frac{12}{x^4} + \frac{1}{x^2} + \frac{1}{5}\sin x$
 q $-4\cos x - \frac{3}{x^{\frac{3}{2}}}$
 r $\frac{5}{6}\sin x + \frac{10}{x^3} - \frac{3}{2x^{\frac{5}{2}}}$
 s $\frac{1}{5}(\cos x + 3\sin x)$
- 2 a 3
 b -1
 c $\frac{1}{2\sqrt{2}}$
 d $2\sqrt{3}$
- 3 a 2
 b $\frac{5}{2}$
 c $-\frac{3}{8}$
 d $3\sqrt{2}$
- 4 -1
- 5 a -1.25
 b -3.03
 c -0.08
 d 4.01
- 6 $x = \frac{\pi}{3}$
 $x = \frac{5\pi}{3}$

- 7 $x = \frac{\pi}{6}$
 $x = \frac{5\pi}{6}$
- 8 $\frac{2\pi}{3}$
- 9 $\left(\frac{\pi}{4}, \frac{1}{2\sqrt{2}}\right)$
- 10 $\left(\frac{2\pi}{3}, 2\sqrt{3}\right)$
 $\left(\frac{4\pi}{3}, -2\sqrt{3}\right)$
- 11 $2\sin\left(\frac{5\pi}{6}\right) - \frac{1}{\left(\frac{5\pi}{6}\right)^2}$
 $1 - \frac{1}{\left(\frac{5\pi}{6}\right)^2}$
 $1 - \frac{36}{25\pi^2}$
 $\frac{25\pi^2 - 36}{25\pi^2}$
 $\frac{(5\pi + 6)(5\pi - 6)}{25\pi^2}$
- 12 a $3\cos x - 2$
 b $x = -\frac{5\pi}{6}$
 $x = -\frac{\pi}{6}$
- 13 $\frac{5\pi}{6}$
- 14 (4.07, 4.14)
- 15 $\left(\frac{4\pi}{3}, 4\right)$
 $\left(\frac{5\pi}{3}, 2\right)$
- 16 $\left(\frac{5\pi}{12}, 5\right)$

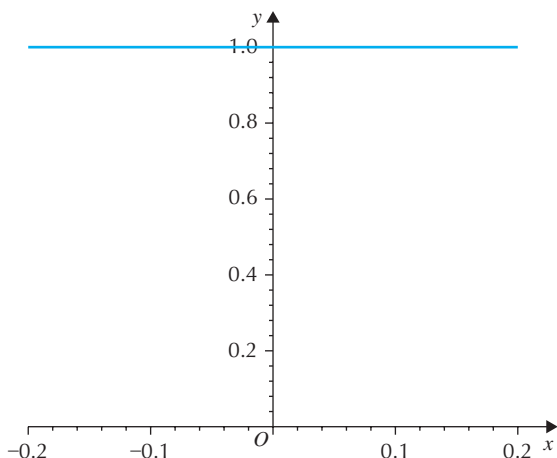
Exercise 9F

1 $\sin(x + h) = \sin x \cos h + \cos x \sin h.$

2



3



4 Very poor and misleading diagram. Degree versus radians issue is a needless obfuscation here.

a $circ = \pi(1)^2$

$$fraction = \frac{h}{360} \pi(1)^2 = \frac{\pi h}{360}$$

b $area = \frac{\pi h}{2\pi} = \frac{h}{2}$

c $OA = 1$

$$OB = 1\cos(h)$$

$$area = \frac{1}{2}(OA)(OB)\sin(h) \\ = \frac{1}{2}\cos(h)\sin(h)$$

d $area = \frac{1}{2}(base)(height)$

$$= \frac{1}{2}(1)1\tan(h) = \frac{1}{2}\tan(h)$$

5 a From above

$$\frac{1}{2}\sin(h)\cos(h) < \frac{h}{2} < \frac{1}{2}\tan(h)$$

b $\frac{2}{\sin(h)\cos(h)} > \frac{2}{h} > \frac{2}{\tan(h)}$

$$\frac{1}{\cos(h)} > \frac{\sin(h)}{h} > \cos(h)$$

c 1

6 a $\frac{\cos(h)-1}{h} \times \frac{\cos(h)+1}{\cos(h)+1} = \frac{(\cos(h))^2-1}{h(\cos(h)+1)}$

$$= \frac{-(\sin(h))^2}{h(\cos(h)+1)}$$

b Yes

$\lim_{h \rightarrow 0}(\sin(h)) = 0$ for obvious reasons. Therefore

$$\lim_{h \rightarrow 0} \left(\frac{-(\sin(h))^2}{h(\cos(h)+1)} \right) = 0$$

7

$$\frac{\sin x(\cos(h)-1) + \cos x \sin(h)}{h} \\ = \sin x \lim_{h \rightarrow 0} \left(\frac{\cos(h)-1}{h} \right) + \cos x \lim_{h \rightarrow 0} \left(\frac{\sin(h)}{h} \right) \\ = \sin x \times 0 + \cos x \times 1 = \cos x$$

8

$$\lim_{h \rightarrow 0} \left(\frac{\cos(x+h) - \cos x}{h} \right) \\ = \lim_{h \rightarrow 0} \left(\frac{\cos x \cos(h) - \sin x \sin(h) - \cos x}{h} \right) \\ = \lim_{h \rightarrow 0} \left(\frac{\cos x(\cos(h)-1) - \sin x \sin(h)}{h} \right) \\ = \cos x \lim_{h \rightarrow 0} \left(\frac{\cos(h)-1}{h} \right) - \sin x \lim_{h \rightarrow 0} \left(\frac{\sin(h)}{h} \right) \\ = -\sin x \lim_{h \rightarrow 0} \left(\frac{\sin(h)}{h} \right) \\ = -\sin x$$

9

$$\frac{d}{dx} \sin x^\circ = \frac{\pi}{180} \cos x^\circ$$

Exercise 9G

1 there is no question here.

a $3(x+4)^2$

b $6(x-2)^5$

c $9(x+3)^8$

d $5(x-1)^4$

e $20(x+1)^3$

f $48(x-3)^5$

g $4(x+5)^7$

h $2(x-5)^6$

i $-\frac{1}{(x+2)^2}$

j $-\frac{4}{(x-5)^5}$

k $-\frac{7}{(x+6)^8}$

l $-\frac{4}{(x-3)^5}$

m $-\frac{12}{(x-2)^4}$

n $-\frac{18}{(x-7)^3}$

o $-\frac{6}{(x+1)^9}$

- p** $-\frac{15}{2(x-4)^{10}}$
q $4(x-1)^3$
r $-5(x+4)^4$
s $18(x+2)^8 - 12x^2$
t $20(x-1)^3 + \frac{3}{\sqrt{x}}$
u $\frac{10}{x^3} - \frac{3}{(x+4)^2}$
v $-\frac{12}{7x^4} - \frac{16}{(x-4)^3}$
- 2 a** $18(3x+1)^5$
b $20(5x-2)^3$
c $10(2x-7)^4$
d $36(4x+1)^8$
e $42(3x-4)^6$
f $180(6x+2)^2$
g $320(5x-4)^7$
h $168(7x-1)^3$
i $-\frac{8}{(4x-1)^3}$
j $-\frac{16}{(2x+5)^9}$
k $-\frac{9}{(9x-2)^2}$
l $-\frac{30}{(5x+4)^7}$
m $-\frac{24}{(2x-1)^5}$
n $-\frac{70}{(7x+1)^3}$
o $-\frac{6}{(2x+5)^4} - 8$
p $-\frac{5}{4x^4} + \frac{12}{(3x-1)^2}$
q $4 - \frac{6}{(x-4)^7} + \frac{1}{x^2}$
r $\frac{2}{x^{\frac{3}{2}}} - \frac{48}{(8x-1)^3}$
s $\frac{3}{2x^{\frac{5}{2}}} + 3\sqrt{x} - \frac{48}{(4x-1)^3}$
t $\frac{4}{x^{11}} + \frac{30}{(3x+4)^6}$
- 3 a** $-5(1-x)^4$
b $-\frac{3}{(x+5)^4}$
c $28(7x+3)^3$
d $4\left(\frac{2x}{3} - 4\right)^5$
e $-30(2-5x)^5$
f $6\left(2 + \frac{3x}{5}\right)^9$
g $-\frac{12}{5}(2-3x)^3$
h $\frac{1}{(6-x)^2}$
i $\frac{30}{(1-2x)^4} - 14x$
j $9\sqrt{x} - \frac{1}{(1-2x)^5}$
- 4 a** $-\frac{4}{(x+1)^5}$
b $-\frac{2}{(x-5)^3}$
c $-\frac{20}{(4x+1)^6}$
d $-\frac{3}{(3x-4)^2}$
e $-\frac{5}{(x-3)^2}$
f $-\frac{6}{(x+1)^4}$
g $-\frac{24}{(2x-5)^3}$
h $-\frac{240}{(5x-1)^7}$
i $-\frac{1}{3(x+2)^2}$
j $-\frac{1}{2(x-1)^3}$
k $-\frac{6}{(x+1)^5}$
l $-\frac{10}{3(3x-2)^6}$
m $\frac{1}{(2-x)^2}$
n $\frac{24}{(5-3x)^2}$
o $\frac{16}{(5-8x)^3}$
p $\frac{9}{(1-2x)^3}$

5 a $\frac{4}{3}(x+5)^{\frac{1}{3}}$

b $\frac{3\sqrt{x-1}}{2}$

c $10(4x+1)^{\frac{3}{2}}$

d $\frac{35}{4}(7x-2)^{\frac{1}{4}}$

e $\frac{1}{3(x-2)^{\frac{2}{3}}}$

f $\frac{1}{2(2x-5)^{\frac{3}{4}}}$

g $-\frac{1}{3(x+4)^{\frac{4}{3}}}$

h $-\frac{15}{2(5x+6)^{\frac{5}{2}}}$

i $-\frac{2}{3(x-1)^{\frac{5}{3}}}$

j $-\frac{3}{8(x+2)^{\frac{5}{2}}}$

k $\frac{2}{3(x-4)^{\frac{1}{3}}}$

l $15(6x+1)^{\frac{3}{2}}$

m $-\frac{1}{2(x+2)^{\frac{3}{2}}}$

n $-\frac{6}{(x-3)^{\frac{7}{4}}}$

o $\frac{1}{5(4-x)^{\frac{6}{5}}}$

p $-45(2-5x)^{\frac{1}{2}}$

q $\frac{6}{(5-2x)^{\frac{7}{4}}}$

6 a $8x(x^2-3)^3$

b $5(3x^2-4x)(x^3-2x^2+1)^4$

c $3(4x^3-5)(x^4-5x-2)^2$

d $-36x(4-3x^2)^5$

e $-\frac{4x+5}{(2x^2+5x-3)^2}$

f $-\frac{4(-2-3x^2)}{(-x^3-2x+3)^5}$

g $-\frac{2x}{(x^2-5)^2}$

h $\frac{4x-1}{2\sqrt{2x^2-x+5}}$

i $\frac{9x^2}{2(2-3x^3)^{\frac{3}{2}}}$

j $6x^3\sqrt{x^4-1}$

k $\frac{3x^2+2x+1}{3(x^3+x^2+x+1)^{\frac{2}{3}}}$

l $\frac{3(\sqrt{x}-2)^5}{\sqrt{x}}$

7 24

8 $-\frac{1}{9}$

9 -192

10 a $x < \frac{1}{4}$

b $-\frac{2}{3}$

c $\frac{3}{16}$

11 $x = -2$

$x = 1$

$x = 4$

12 $(-3, 2)$

13 a $x^2 - x + 3 = (x - \frac{1}{2})^2 + \frac{11}{4} > 0$
therefore square root is real for all x .

b $x = 3$

14 a $3(px^2 - 4x + p)^2(2px - 4)$

b If $p = 2$ then root $x = 1$

If $p = -2$ then root $x = -1$

15 a $g'(x) = -\frac{2(p-5)x}{((p-5)x^2 - 9p + 2)^2}$

b $p = 5$

16 a $(-1)^3 + (-1)^2 - 4(-1) - 4 = 0$

$(x+2)(x-2)(x+1)$

b $-\frac{12x^3 + 12x^2 - 48x - 48}{(3x^4 + 4x^3 - 48x)^2}$

c $-2, -1, 2$

Exercise 9H

- 1 a $2\cos 2x$
 b $-5\sin 5x$
 c $12\cos 4x$
 d $-18\sin 3x$
 e $-2\sin\left(2x + \frac{\pi}{6}\right)$
 f $\cos(6x - \pi)$
 g $6\cos(4x)$
 h $-6\sin(10x)$
 i $2\cos(6x + 2)$
 j $2\sin\left(\frac{x}{4}\right)$
 k $-\cos(2 - x)$
 l $3\cos(1 - 9x)$
 m $-6\cos(3x) - 5\sin(5x)$
 n $\cos x - 24\sin(3x)$
 o $3\cos 3x + 2\sin x$
 p $\frac{5}{2}\cos(2x - \pi) + \frac{3}{2}\sin\left(\frac{3x}{2}\right)$

- 2 a 1
 b -1
 c $-\frac{3\sqrt{3}}{2}$
 d $\sqrt{2}$
 e 6

- 3 a 1.42
 b -8.16
 c -5.75
 d 1.68

- 4 a $2\cos x \sin x$
 b $-3\sin x (\cos x)^2$
 c $6\cos x (\sin x)^2$
 d $-30\sin x (\cos x)^5$
 e $-2\cos x (\sin x)^3$
 f $-6\sin x (\cos x)^4$
 g $3(\cos x - \sin x)(\cos x + \sin x)^2$
 h $10\cos(1 - 2x)(\sin(2x - 1))^4$

5 1

- 6 a $-\frac{\cos x}{\sin x} \frac{1}{\sin x} = -\frac{\cos x}{(\sin x)^2}$
 b $3\frac{\tan x}{\cos x}$

c $\frac{\cos x}{2\sqrt{\sin x}}$

d $2\frac{\tan x}{(\cos x)^2}$

e $-3\frac{\tan 3x}{\cos 3x}$

f $\frac{\cos x}{2\sqrt{\sin x}}$

g $\cos x \cos(\sin x)$

h $-\cos x \sin(\sin x)$

7 $y = \sin x^\circ = \sin\left(\frac{\pi}{180}x\right)$

$$\frac{dy}{dx} = \frac{\pi}{180} \cos x^\circ$$

8 $y = \cos x^\circ = \cos\left(\frac{\pi}{180}x\right)$

$$\frac{dy}{dx} = -\frac{\pi}{180} \sin x^\circ$$

9 $x = \frac{2\pi}{9}$

$$x = \frac{4\pi}{9}$$

$$x = \frac{8\pi}{9}$$

10 $\left(\frac{\pi}{12}, 2\sqrt{3}\right)$

$$\left(\frac{5\pi}{12}, -2\sqrt{3}\right)$$

11 $x = \frac{7\pi}{18}$

$$x = \frac{11\pi}{18}$$

12 $x = \frac{\pi}{2}$

$$x = \frac{7\pi}{6}$$

$$x = \frac{3\pi}{2}$$

$$x = \frac{11\pi}{6}$$

13 a $\cos(3x^2 - 1)$

b $6x$

c does equal $\frac{3}{2}(\cos 2x + 1)$

d $y = 3(\cos x)^2 - 1$

also

$$y = \frac{a}{\sqrt{3}}x$$

so $a = 4.5$

14 $\frac{\pi}{6}$